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COROTATION DERIVATIVES AND DEFINING RELATIONS IN THE THEORY OF LARGE PLASTIC STRAINS

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UDC 539.374

Interest in elastoplasticity with large strains, by which here we shall mean deformations with strain gradients exceeding (componentwise) 0.1, has increased markedly in the last decade. The main problem addressed by the theory of large elastoplastic strains is the derivation of the defining relations, in whose formulation some types of objective differential measures of the stressed and strained states, called corotational in the literature published abroad, are widely used. In this paper corotational derivatives are defined in a unified manner, and A. A. Il'yushin's theory of elastoplastic processes is extended to the case of large plastic deformations.

1. In what follows we shall require indifferent tensors. For this, following [1], we introduce two motions $r(\xi^i, t)$ and $r'(\xi^i, t)$ of the volume of the continuous medium under study differing by a rigid displacement:

$$\mathbf{r}'(\xi^i, t) = \mathbf{p}'(t) + [\mathbf{r}(\xi^i, t) - \mathbf{p}(t)]\cdot\mathbf{O}(t). \quad (1.1)$$

Here $p(t)$ is the radius vector of a particle, chosen as the pole, in the motion $r(\xi^i, t)$, $p'(t)$ is the pole in the motion $r'(\xi^i, t)$; $O(t)$ is a properly orthogonal tensor; (ξ^i, t) are Lagrangian variables. We shall denote the reference configuration by $\mathscr{K}_0$, and the actual configuration in the motions r and r' by $\mathscr{K}_t$ and $\mathscr{K}_t'$, respectively. The basis vectors in $\mathscr{K}_0$ are $\overset{\circ}{e}_i = \partial R_0/\partial\xi^i$ and the basis vectors in $\mathscr{K}_t$ and $\mathscr{K}_t'$ are

$$\widehat{\mathbf{e}}_i = \frac{\partial\mathbf{r}}{\partial\xi^i},\quad \widehat{\mathbf{e}}_i' = \frac{\partial\mathbf{r}'}{\partial\xi^i},\quad i = \overline{1, 3}.$$

From (1.1)

$$\widehat{\mathbf{e}}_i' = \widehat{\mathbf{e}}_i\cdot\mathbf{O} = \mathbf{O}^T\cdot\widehat{\mathbf{e}}_i,\quad \widehat{\mathbf{e}}_i = \widehat{\mathbf{e}}_i'\cdot\mathbf{O}^T = \mathbf{O}\cdot\widehat{\mathbf{e}}_i'.$$

Analogous relations also follow from the properties of the orthogonal tensor for the vectors of the conjugate basis:

$$\widehat{\mathbf{e}}'^i = \widehat{\mathbf{e}}^i\cdot\mathbf{O} = \mathbf{O}^T\cdot\widehat{\mathbf{e}}^i,\ \widehat{\mathbf{e}}^i = \widehat{\mathbf{e}}'^i\cdot\mathbf{O}^T = \mathbf{O}\cdot\widehat{\mathbf{e}}'^i.$$

Perm'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 160-166, March-April, 1987. Original article submitted February 12, 1986.

0021-8944/87/2802-0311$12.50

An arbitrary tensor $Q = \hat{Q}^{i_1 \cdots i_n} \hat{e}_{i_1}' \ldots \hat{e}_{i_n}'$ of rank n is said to be indifferent if in the motions r and r' the components of the tensor in the bases $\hat{e}_i$ and $\hat{e}_i'$ do not change:

$$Q'(r') = \hat{Q}'^{i_1 \ldots i_n}(r') \hat{e}'_{i_1} \ldots \hat{e}'_{i_n} = \hat{Q}^{i_1 \ldots i_n}(r) \hat{e}_{i_1} \cdot O \ldots \hat{e}_{i_n} \cdot O \quad \forall t. \tag{1.2}$$

The tensor Q itself is, of course, different for the indicated motions, $Q'(r') \neq Q(r)$. For a double tensor, for example, from (1.2) $Q'(r') = O^T \cdot Q(r) \cdot O$.

We also introduce the concept of a tensor invariant with respect to rigid motion: an arbitrary tensor Q of rank n is said to be invariant under rigid motion if its components in the basis $\mathring{e}_i$ in the motions r and r' are identical for $\forall t$:

$$\mathring{Q}'^{i_1 \ldots i_n}(r') = \mathring{Q}^{i_1 \ldots i_n}(r). \tag{1.3}$$

The equality $Q'(r') = Q(r)$ follows from (1.3) by virtue of the invariance of the basis in $\mathscr{K}_0$.

Indifferent tensors include, for example, Almansi's strain tensor A, the right distortion tensor V, the logarithmic measure of strain $\hat{H} = \ln V$, and Cauchy's (Euler's) stress tensor σ. The Cauchy–Green stress tensor C the left distortion tensor U, the logarithmic measure of strain $\mathring{H} = \ln U$, and the second Piol–Kirchhoff tensor are invariant with respect to rigid motion.

Derivatives of tensors — measures of the stressed and strained states — are usually employed in the formulation of the defining relations of the theory of plasticity. Evidently, the total (substantial) derivatives of the invariant tensors presented above are also invariant under rigid motion. It is not difficult to show that the substantial derivatives of indifferent tensors are not indifferent. Because of this indifferent derivatives of indifferent tensors, which include also some corotational derivatives, are widely employed in the mechanics of continuous media.

It should be noted that in most works devoted to geometrically nonlinear problems different indifferent derivatives are introduced in a formal manner, without any explanation of their physical meaning. References [1-4] are an exception in this respect. In [1-3] the physical meaning of Jaumann's derivative is studied, and in [4] Jaumann's, Oldroyd's, Cotter and Rivlin's, and some other derivatives are studied. However objective derivatives in the works cited are defined in terms of the components of tensors. This, first of all, complicates the derivations; second, certain assumptions are required for the derivations (for example, the instantaneous and reference Lagrangian and Euler coordinate systems coincide at the moment under study); and, third, it often leads to an incomplete interpretation of the objective derivatives of tensors as the derivatives of components of the tensor under study. In this paper the corotational derivative tensors are introduced as the relative rates of change of the tensors defined by an observer in a moving coordinate system. At the same time all derivations are made in the tensor (symbolic, component-free) form, which substantially simplifies the calculations and eliminates the need for introducing the assumptions mentioned above.

Consider an arbitrary particle $\mathscr{M}$ of the continuous medium together with a small neighborhood of the particle, whose state may be regarded as uniform. We associate with the particle a moving coordinate system $\mathscr{M}\zeta^1\zeta^2\zeta^3$ with the basis q_i (the conjugate basis q^i), and we assume that under rigid motion of the particle (together with the small neighborhood) the coordinate system $\mathscr{M}\,\zeta^1\zeta^2\zeta^3$ undergoes an identical motion. Let the vector a and tensor Q the relative rates a^r and Q^r, according to the well-known definition in general mechanics, can be expressed as follows:

$$a^r(q_i) = \left[\frac{d}{dt}(a \cdot q^i)\right] q_i = (\dot{a} \cdot q^i + a \cdot \dot{q}^i)\, q_i; \tag{1.4}$$

$$Q^r(q_i) = (q^i \cdot \dot{Q} \cdot q^j + \dot{q}^i \cdot Q \cdot q^j + q^i \cdot Q \cdot \dot{q}^j) q_i q_j; \tag{1.5}$$

$$a^r(q^i) = (\dot{a} \cdot q_i + a \cdot \dot{q}_i) q^i; \tag{1.6}$$

$$Q^r(q^i) = (q_i \cdot \dot{Q} \cdot q_j + \dot{q}_i \cdot Q \cdot q_j + q_i \cdot Q \cdot \dot{q}_j) q^i q^j. \tag{1.7}$$

In the case of a rigid Cartesian orthogonal system $\mathscr{M}\zeta^1\zeta^2\zeta^3$ the difference between (1.4) and (1.6), (1.5) and (1.7) vanishes.

The relations (1.4)-(1.7) enable the derivation of the well-known corotational derivatives. We shall study some of them. Let the moving system be a Lagrangian comoving system

$\mathscr{M}\zeta^1\zeta^2\zeta^3$ with the basis $\hat{e}_i$. It can be shown that

$$\dot{\hat{\mathbf{e}}}_i = \hat{\mathbf{e}}_i \cdot \hat{\nabla}\mathbf{v} = \hat{\nabla}\mathbf{v}^T \cdot \hat{\mathbf{e}}_i; \tag{1.8}$$

$$\dot{\hat{\mathbf{e}}}^i = -\hat{\nabla}\mathbf{v} \cdot \hat{\mathbf{e}}^i = -\hat{\mathbf{e}}^i \cdot \hat{\nabla}\mathbf{v}^T, \tag{1.9}$$

where v is the velocity vector; $\hat{\nabla}(\cdot) = \hat{e}^i[\partial(\cdot)/\partial\xi^i]$ is Hamilton's operator (nabla). Using (1.9), from (1.4) and (1.5) we obtain the so-called Oldroyd derivative [5] of a vector and rank-2 tensor:

$$\mathbf{a}^{Ol} = \dot{\mathbf{a}} - \mathbf{a} \cdot \hat{\nabla}\mathbf{v} = \dot{\mathbf{a}} - \hat{\nabla}\mathbf{v}^T \cdot \mathbf{a}, \ \mathbf{Q}^{Ol} = \dot{\mathbf{Q}} - \hat{\nabla}\mathbf{v}^T \cdot \mathbf{Q} - \mathbf{Q} \cdot \hat{\nabla}\mathbf{v}.$$

Relating the mobile coordinate system with the conjugate basis vectors $\hat{e}^i$, from (1.8), (1.6), and (1.7) we determine the corresponding relative rates of change of the vector and tensor, called the Cotter and Rivlin derivatives [6]:

$$\mathbf{a}^{CR} = \dot{\mathbf{a}} \cdot \hat{\nabla}\mathbf{v} \cdot \mathbf{a} = \dot{\mathbf{a}} + \mathbf{a} \cdot \hat{\nabla}\mathbf{v}^T, \ \mathbf{Q}^{CR} = \dot{\mathbf{Q}} + \hat{\nabla}\mathbf{v} \cdot \mathbf{Q} + \mathbf{Q} \cdot \hat{\nabla}\mathbf{v}^T.$$

We note that the tensor of velocity strain $D = \frac{1}{2}(\hat{\nabla}v + \hat{\nabla}v^T)$ is a Cotter and Rivlin derivative of Almansi's strain tensor: $D = A^{CR}$. It can also be shown that the well-known [1] Truesdell derivative of the Cauchy stress tensor

$$\boldsymbol{\sigma}^{Tr} = \dot{\boldsymbol{\sigma}} - \hat{\nabla}\mathbf{v}^T \cdot \boldsymbol{\sigma} - \boldsymbol{\sigma} \cdot \hat{\nabla}\mathbf{v} + \boldsymbol{\sigma}(\hat{\nabla} \cdot \mathbf{v})$$

is the same (up to a scalar factor) as Oldroyd's derivative of the Kirchhoff tensor $t = (\mathring{\rho}/\hat{\rho})\sigma$, i.e.,

$$\boldsymbol{\sigma}^{Tr} = \frac{\hat{\rho}}{\mathring{\rho}} \mathbf{t}^{Ol}$$

($\mathring{\rho}$ and $\hat{\rho}$ are the density of the medium in $\mathscr{K}_0$ and $\mathscr{K}_t$).

The derivatives presented above have the drawback, in our opinion, that the coordinate systems with respect to which these derivatives determine the rate of change of the tensors are deformable. Obviously, deformations of the comoving system itself must also be taken into account.

It is probably because of this that the rates of change of tensors with respect to rigid undeformable coordinate systems are most widely used. Let $\mathscr{M}\zeta^1\zeta^2\zeta^3$ be a Cartesian orthogonal coordinate system with the orthonormal basis $q_i = q^i$, in translational motion with the velocity of the particle $\mathscr{M}$ and rotating as a rigid body. Generally speaking, there is an infinite number of ways to choose the last motion. At the same time there is an infinite number of representations of the motion of a particle of a continuous medium by a combination of translational and rotational motions as a rigid whole (we shall call this part of the motion "quasirigid" and identify it with the motion of the coordinate system $\mathscr{M}\zeta^1\zeta^2\zeta^3$) and a deformation motion. After the quasirigid motion is chosen the deformation part is determined uniquely.

Assuming that at each moment the system $\mathscr{M}\zeta^1\zeta^2\zeta^3$ rotates with the angular velocity of the material fibers, which coincide with the principal axes of the tensor D [characterized by the vorticity tensor $W = \frac{1}{2}(\hat{\nabla}v^T - \hat{\nabla}v)$], so that $\dot{q}_i = \dot{q}^i = W \cdot q_i = -q_i \cdot W$, we arrive at the relative velocity called the Jaumann–Noll derivative [1]:

$$\mathbf{a}^J = \dot{\mathbf{a}} + \mathbf{a} \cdot \mathbf{W} = \dot{\mathbf{a}} - \mathbf{W} \cdot \mathbf{a}; \tag{1.10}$$

$$\mathbf{Q}^J = \dot{\mathbf{Q}} - \mathbf{W} \cdot \mathbf{Q} + \mathbf{Q} \cdot \mathbf{W}. \tag{1.11}$$

Let the quasirigid instantaneous rotation be characterized by the spin tensor Ω (determining the angular rotational velocity of the principal axes of the right distortion tensor V relative to the principal axes of the left distortion tensor U). Then the relative rate of change is characterized by the often-used Zaremba's corotational derivative [7-9 and others].

$$\mathbf{a}^Z = \dot{\mathbf{a}} + \mathbf{a} \cdot \boldsymbol{\Omega} = \dot{\mathbf{a}} - \boldsymbol{\Omega} \cdot \mathbf{a}; \tag{1.12}$$

$$\mathbf{Q}^Z = \dot{\mathbf{Q}} - \boldsymbol{\Omega} \cdot \mathbf{Q} + \mathbf{Q} \cdot \boldsymbol{\Omega}. \tag{1.13}$$

Analogous corotational derivatives can be constructed, identifying the instantaneous motion of the rigid system $\mathscr{M}\zeta^1\zeta^2\zeta^3$ with the motion of the principal axes of the right distortion tensor.

The corotational derivatives (1.10)-(1.13), as compared with the Oldroyd and Cotter–Rivlin derivatives, have two remarkable properties: a) the corotational derivatives of the scalar product of rank-2 tensors **A** and **B** and of a rank-2 tensor by a vector are determined by the usual rules of differentiation, for example, $(\mathbf{A}\cdot\mathbf{B})^Z = \mathbf{A}^Z\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{B}^Z$; b) in the case that the corotational derivative of a vector or rank-2 tensor equals the zero tensor, their invariants at a given moment in time are stationary. Corotational derivatives can be defined, generally speaking, for any tensors.

As pointed out in [10, 11], useful measures of strain and the rate of strain are the Henke's strain tensor and its corotational derivatives. We shall present expressions for $\hat{H}^Z$, $\hat{H}^J$, whose derivation is given in greater detail in [10, 11]:

$$\hat{\mathbf{H}}^Z = \mathbf{D} + \mathbf{L}_Z,\ \hat{\mathbf{H}}^J = \mathbf{D} + \mathbf{L}_J; \tag{1.14}$$

$$\mathbf{L}_Z = \sum_{\substack{i,k=1\\ i\neq k}}^{3}\left\{\left[\frac{2V_kV_i}{V_k^2-V_i^2}\ln\left(\frac{V_k}{V_i}\right)-1\right](\hat{\mathbf{p}}^i\cdot\mathbf{D}\cdot\hat{\mathbf{p}}^k)\,\hat{\mathbf{p}}_k\hat{\mathbf{p}}_i\right\}, \tag{1.15}$$

$$\mathbf{L}_J = \sum_{\substack{i,k=1\\ i\neq k}}^{3}\left\{\left[\frac{V_k^2+V_i^2}{V_k^2-V_i^2}\ln\left(\frac{V_k}{V_i}\right)-1\right](\hat{\mathbf{p}}^i\cdot\mathbf{D}\cdot\hat{\mathbf{p}}^k)\,\hat{\mathbf{p}}_k\hat{\mathbf{p}}_i\right\},$$

$\hat{p}_i = \hat{p}^i$ are the principal vectors; V_i are the principal values of the right measure of distortion; $\mathscr{L}_Z^{ik} = 0$, $\mathscr{L}_J^{ik} = 0$ for $V_i = V_k$ [10, 11]. It is not difficult to see that $\hat{H}^Z$, $\hat{H}^J$ are determined completely by the velocity strain tensor, the position of the principal axes, and the principal values of the tensor **V**. The first invariants of $d\hat{H}/dt$, D and characterize the rate of change of the volume.

2. In existing studies [12, 13, etc.] the relations of the theory of flow with the stress and strain transformation tensors replaced by the Jaumann–Noll derivative of the Cauchy stress tensor and the velocity strain tensor, respectively, are usually employed as the defining relations. As shown in [7], however, even when the model of a gyroelastic material is employed such relations lead to oscillations of the stresses. The conclusion that the Jaumann–Noll derivative is unsuitable and that a derivative of the type (1.3) is suitable for the formulation of the defining relations, drawn based on the solution of the problem of simple shear using instead of σ^J the corotational derivative σ^Z, appears to be unjustified.

The appearance of oscillations in the determining relations of the type indicated is best explained, in our opinion, by the fact that the corotational derivatives of the measure of the stressed state do not correspond to those of the measure of the strained state. Indeed, as shown above, σ^J is the rate of change of the Cauchy stress tensor, measured by an observer rotating together with the rigid coordinate system with an angular velocity corresponding to the tensor **W**. At the same time the velocity strain tensor **D**, representing in the defining relations under analysis the velocity of the measure of the strained state, is the rate of change of Almansi's tensor, fixed by an observer in a deformable comoving Lagrangian coordinate system with the basis $\hat{e}^i$.

The foregoing physically justifies the use of corotational derivatives of one type as measures of both the stressed and strained states in the formulation of the defining relations. The use of different types of corotational derivatives probably makes it necessary to introduce additional terms into the defining relations, taking into account the difference of the motions of the moving coordinate systems. For the above-mentioned model problem of the simple shear of a strip of quasielastic material the solution in which the Jaumann–Noll derivative of Alamansi's tensor A^J is employed for the velocity of the measure of the strained state instead of $D = A^{CR}$ does not contain oscillating terms.

Aside from the indicated deficiency of the existing defining relations, it should be noted that the theory of flow is applicable only to a limited class of plastic strain processes along trajectories of small curvature, even for small strains [14]. At the same time, a theory of elastoplastic processes has been developed for small strains and is now generally accepted

[15]. In this connection, an attempt was made to extend the theory of elastoplastic processes to the case of large plastic strains.

One of the basic concepts in the theory of elastoplasitic processes is the concept of the image of the loading process [15]. Here, in order to construct the strain trajectory of the particle under study at each moment in time, it is necessary to study the strain of the same triplet of material fibers. Analysis of possible methods for decomposing the motion into quasirigid motion is separated with the help of an orthogonal Cartesian moving coordinate system, $\mathscr{M}\zeta^1\zeta^2\zeta^3$, rotating at each moment in time with an angular velocity corresponding to the spin tensor Ω, and then the strain part is described by Zaremba's derivative of the strain tensor. The latter is described by the logarithmic measure of strain H. The strain trajectory and the stress vector [15] are constructed at each moment in time according to the components of $\hat{H}$ and σ in the basis q^i of the moving system $\mathscr{M}\zeta^1\zeta^2\zeta^3$. It can be shown that the determining relations in this case have the quite general form

$$\sigma^Z = \mathbf{F} : \hat{\mathbf{H}}^Z + \mathbf{R}, \tag{2.1}$$

where F is a quadruple tensor of properties (different for different particular theories); R is a double tensor, which does not depend on $\hat{H}^Z$, and, $\hat{H}^Z$ is determined by (1.14) and (1.15). At the same time all tensors appearing in (2.1) are indifferent. Expressions for the tensors F and R for different particular theories are presented in [10]. For example, for strain along trajectories with average curvature the tensor R is zero, and

$$\mathbf{F} = \frac{\Phi'(s) - k(s)\sigma_i(s)}{\sigma_i^2(s)}\mathbf{SS} + \frac{\sigma_i(s)k(s)}{2}(\mathbf{C}_{II} + \mathbf{C}_{III}) + \frac{K - \sigma_{II}(s)k(s)}{3}\mathbf{C}_I.$$

Here $\Phi(s)$ is a universal function, characterizing the scalar properties of the material; $\sigma_i(s)$ is the intensity of the stresses; S is the deviator of the stress tensor; $\Phi'(s) = \partial\Phi(s)/\partial s$; $\sigma_i(s)$ are isotropic tensors of rank 4 [1]; K is the bulk modulus; k(s) is a function characterizing the change in the angle of approach (the angle between the stress vector and the tangent to the strain trajectory); and, s is the arc length along the strain trajectory.

$$s = \int_0^t (\hat{\mathbf{h}}^Z : \hat{\mathbf{h}}^Z)^{\frac{1}{2}} d\tau, \quad \hat{\mathbf{h}}^Z = \hat{\mathbf{H}}^Z - \frac{1}{3} I_1(\hat{\mathbf{H}}^Z)\mathbf{E}.$$

An alternative to the foregoing approach could be the employment of invariant tensors and their substantial derivatives (also invariant) for the generalization of the theory of elastoplastic processes. In this case the defining relations are most efficiently constructed using the Lagrangian approach. The advantage of this approach is that the question of the nonuniqueness of the decomposition of the motion of a deformable medium into quasirigid motion and strain does not arise. At the same time there is no need to use corotational derivatives, since when invariant tensors are used a rigid motion does not change them. A final judgement about the usefulness of a defining relation of the form (2.1) can be made only based on the results of experiments on complex loading with large plastic strains. Unfortunately at the present time there are no such data. It should be noted that such data can be obtained only based on a preliminary theoretical analysis of the possible types of defining relations.

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GENERAL ENERGY RELATIONS FOR RAIL GUNS

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UDC 533.95:538.4

The question of energy relations is one of the most important questions in the description of the operation of rail guns and the analysis of their potential possibilities. For limited energy storage in the supply source, the efficiency of the conversion of electromagnetic energy into kinetic energy of the accelerated body essentially determines the possible scale of the experiment. There are a large number of papers devoted to this problem (see, e.g., [1-9]). In the analysis of the energy conversion efficiency there arises the question of the optimal choice of the parameters of the system under study, ensuring the most efficient conversion of energy stored in the source into the kinetic energy of the accelerated body: the energy of the source, the inductance and active resistance of the circuit, the inductance per unit length of the accelerator, the mass of the body, and other parameters characterizing only the given type of source. Since the number of determining parameters is small, the main method reduced to the numerical solution of the equations determining the acceleration process together with analytical study of the asymptotic states. This approach is developed in particular in the works cited above. Numerical solutions permit determining the optimal parameters of the system for a given energy source, but do not fully reveal the general energy relations, characteristic of the rail gun method for accelerating solids.

The main analytical solutions, which permit evaluating, under a number of assumptions, the velocity and efficiency with which the energy stored in the source is converted into kinetic energy of the body, were obtained in [1-3, 5, 6]. Without examining in detail the assumptions made by the authors, we point out only that an assumption common to all of them is that the active resistance of the circuit equals zero. As a consequence of this assumption it was concluded that with a correct choice of the parameters of the accelerator and source the conversion efficiency can be close to unity. At the same time the results of a numerical solution of the equations and qualitative considerations regarding the ohmic losses show that the effect of the active resistance of the circuit on the velocity of the body and the efficiency can be very significant [5-8]. In [7, 8] it is pointed out that ohmic losses in real accelerators can substantially limit the possibility of rail guns.

The most complete analysis of the energy distribution in a circuit consisting of a capacitor bank and a rail gun has apparently been carried out in [9] for the experiments described in the literature. For the acceleration of particles with a mass of 0.3 g a maximum velocity of 3.3 km/sec with an energy conversion efficiency of ~1% was achieved. The ratio of the kinetic energy of the body to the ohmic energy dissipated in the plasma bridge, accelerating a dielectric body, equalled approximately 13%. Data which would permit evaluating the ratio

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 166-171, March-April, 1987. Original article submitted May 21, 1986.

 0021-8944/87/2802-0316$12.50